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*Published in:*  
Physical Review B

*Link to article, DOI:*  
[10.1103/PhysRevB.37.2278](https://doi.org/10.1103/PhysRevB.37.2278)

*Publication date:*  
1988

*Document Version*  
Publisher's PDF, also known as Version of record

[Link back to DTU Orbit](#)

*Citation (APA):*  
Mouritsen, O. G., & Præstgaard, E. (1988). Reply to "Domain-growth kinetics of systems with soft walls". *Physical Review B*, 37(4), 2278-2279. <https://doi.org/10.1103/PhysRevB.37.2278>

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## Reply to "Domain-growth kinetics of systems with soft walls"

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(Received 8 May 1987)

On the basis of computer-simulation results for three different models with soft domain walls it is argued that the zero-temperature domain-growth kinetics falls in a separate universality class characterized by a kinetic growth exponent  $n \approx 0.25$ . However, for finite temperatures there is a distinct crossover to Lifshitz-Allen-Cahn kinetics,  $n = 0.50$ , thus suggesting that the soft-wall and hard-wall universality classes become identical at finite temperatures.

In their Comment on the work by one of us<sup>1</sup> on the domain-growth kinetics of systems with soft domain walls, van Saarloos and Grant (vSG)<sup>2</sup> raise basically two points: (i) rescaling of the time scale used in our zero-temperature computer simulations resulting in logarithmically slow growth, and (ii) possible crossover to Lifshitz-Allen-Cahn<sup>3</sup> kinetics at finite temperatures.

In response to these points we argue in this reply, on the basis of comparative model studies<sup>4-6</sup> and recent unpublished results,<sup>7</sup> the following.

(i) There are no indications of logarithmically slow zero-temperature growth in simulation studies of soft-wall models. All models, independent of the details, lead to the same zero-temperature kinetic exponent,  $n \approx 0.25$ . At late times, the growth may be slowed down due to finite-size-induced slab effects wellknown from Ising models.<sup>8</sup> We suggest that the prediction of vSG may be obscured by the one-dimensional nature of their calculation. Furthermore, we argue that one of the assumptions underlying the Allen-Cahn law may not hold for the soft-wall models at zero temperature.

(ii) Recent results on the finite-temperature kinetics of the soft-wall models<sup>7</sup> indeed indicate that the Allen-Cahn law is recovered at finite temperatures, in agreement with our previous results for the herringbone model,<sup>5</sup> the study by Milchev, Binder, and Heermann<sup>9</sup> of the  $\phi^4$  model, and the experimental study pointed out by vSG of the growth kinetics in smectic films.<sup>10</sup>

Three two-dimensional soft-wall models with continuous single-site variables (spins or rotors) will be referred to in the following. They are conveniently labeled by their number  $p$  of degenerate ordered ground states: the  $p = 2$  model,<sup>1,2</sup> the  $p = 4$  model<sup>4</sup> and the  $p = 6$  herringbone model.<sup>5</sup> vSG concentrate on the  $p = 2$  model. Concerning point (i), they study the model in a simple limit where it reduces to a one-dimensional problem. While their arguments about the  $\ln t$  rescaling of the Monte Carlo time scale imposed by the particular dynamics employed in Ref. 1 are undoubtedly correct in *one dimension* as well as for certain special geometries of the domain-wall network (we have verified the slowing down of growth for linear domain walls along the lattice directions<sup>7</sup>), we do not be-

lieve that the argument holds for a random *two-dimensional* network with curvature. It is essential for the domain-wall motions that the model Hamiltonian contains the original next-nearest-neighbor couplings<sup>1</sup> which are disregarded in the calculation by vSG. In fact, we find that the argument of vSG is pertinent for slab configurations. The slowing-down effects due to finite-size-induced slab configurations also inhibit the growth in the soft-wall models at late times, quite similar to what is found in Ising models.<sup>8</sup> Extensive studies of all three soft-wall models at zero-temperature show no signs of slowing down in the time regime where finite-size effects have been eliminated,<sup>1,4,5,7</sup> and the striking result is that the same value of the growth exponent,  $n \approx 0.25$ , is found in all cases studied. Moreover, the wide-wall limit of anisotropic high- $p$  Potts models gives the same result.<sup>6</sup>

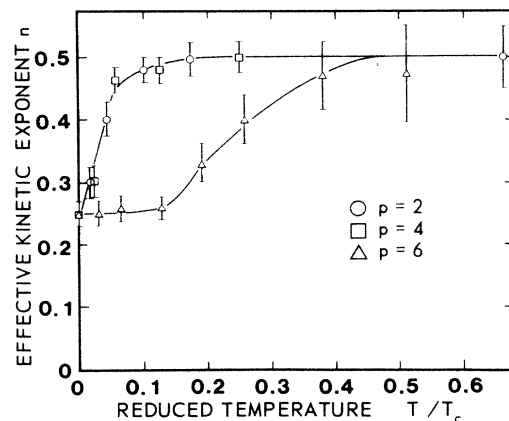


FIG. 1. Effective late-time growth exponents vs reduced quenching temperature  $T/T_c$  for three different soft-wall models.  $p$  denotes the number of degenerate ordered domains of the model. The solid lines are drawn as guides to the eye. The late-time region in the computer simulations is taken as the time range over which an asymptotic growth behavior has established itself in systems containing up to  $200^2$  particles. The time of crossover  $t_c$  to the late-time behavior depends on the model,  $t_c$  being smaller the smaller the value of  $p$ .

Turning now to point (ii), our results from finite-temperature growth kinetics of all three soft-wall models<sup>5,7</sup> indeed show (Fig. 1), in accordance with the suggestion by vSG, that there is a crossover to the Allen-Cahn law with  $n=0.50$ , independent of the actual degree of the domain-wall softness. These findings indicate that the hard-wall and soft-wall domain-growth kinetics fall in the same universality class at finite temperatures. This is moreover consistent with the finite-temperature Allen-Cahn kinetics found for the soft-wall  $\phi^4$  model<sup>9</sup> and interestingly enough with the finite-temperature electric-field quenching experiments on smectic films by Pindak, Young, Meyer, and Clark.<sup>10</sup> These experiments even showed that the kinetic exponent is independent of the width of the wall.

The question remains: What causes the difference between the hard-wall and soft-wall kinetics at zero temperature? We believe that a key to answering this question may be the observation that the  $n=0.25$  exponent for the various soft-wall models is found to be independent of the actual degree of softness or wall thickness. Thus it appears that the relevant property of the soft-wall models is not that the walls are soft as such and how soft they are, but rather their capacity of softening, in particular soften-

ing locally, in response to high curvature. This observation hints at a possible breakdown of the basic assumption underlying the Allen-Cahn theory<sup>3,2</sup> regarding the inverse proportionality of wall width and surface tension. Inspection of snapshots of domain-wall networks generated in computer-simulation studies of the soft-wall models<sup>7</sup> shows that, in regions of high curvature the walls not only move but also soften, thus impeding the growth. These observations call for a new theory of zero-temperature soft-wall domain-growth kinetics which accounts for the additional effect of domain-wall softening in response to curvature.

Finally, it is now obvious that the  $T=0$  limit may not be very interesting in an experimental context as far as the asymptotic growth behavior is concerned. Still, it represents a well-defined mathematical limit whose properties may, as Fig. 1 suggests, have influence on the crossover behavior at low but finite temperatures.

Stimulating discussions on the present subject with Gary Grest are gratefully acknowledged. This work was supported by the Danish Natural Science Research Council under Grant J. No. 5.21.99.72.

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